Conference on Functional Analysis and Fractals

(February 16-18, 2024)

Titles and Abstracts

Department of Applied Sciences
IIIT Allahabad

Commutant lifting theorem on the polydisc

Prof. Jaydeb Sarkar
ISI Bangalore

Abstract. Sarason's one variable commutant lifting theorem is a key result in the theory of linear analysis, complex analysis, and Hilbert function space theory, which has a stellar reputation in its application to classical results like Nevanlinna-Pick interpolation, Caratheodory-Fejer interpolation problem, Nehari interpolation problem, von Neumann inequality, isometric dilations, just to name a few.

We shall commence our discussion by providing a historical overview of the above themes, spanning a century. In the second half of the talk, we will go over some recent advances in the commutant lifting theorem on the polydisc and its applications to interpolation and perturbation problems.

Complex Continued Fractions and the Symmetric Hausdorff Dimension Spectrum

Prof. Roger D. Nussbaum
Rutgers University, USA

Abstract. Let $(script)B := \{m+in = b : m \text{ a natural number and } n \text{ an integer}\}$. For a complex number z and for b in $(script)B$ we define a map $(\theta)_b : H := \{z : Re(z) \geq 0\} \to H$ by $(\theta)_b(z) = 1/(z+b)$. For a nonempty set F contained in $(script)(B)$ there is a unique, nonempty "invariant set" $E(F)$ in H such $E(F)$ equals the union of $(\theta)_b(E(F))$ for b an element of F. If F is finite, $E(F)$ is compact, but generally $E(F)$ is not compact if F is infinite. We write $\dim(E(F)) := \dim(F) :=$ the Hausdorff dimension of $E(F)$. In recent work, Chousionis, Leykekhman and Urbanski (abbreviated C-L-U) prove that for every s with $0 \leq s \leq \dim((script)B)$, there exists a set F contained in (script)B with $\dim(F) = s$.

For a complex number z, we write $conj(z)$ to denote the complex conjugate of z and we say that a set T in the complex plane is conjugate symmetric if z is an element of T iff $conj(z)$ is an element of T. Since $(script)B$ is conjugate symmetric, it is natural to ask whether for each s with $0 \leq s \leq \dim((script)B)$ there exists a conjugate symmetric set F contained in $(script)(B)$ (so $E(F)$ is conjugate symmetric) with the property that $\dim(F) = s$. This seems more difficult than the original C-L-U question and our methods are different. The answer is positive and we shall indicate some ingredients of the proof and mention a theoretical algorithm for finding F, given s. Time permitting, we shall mention analogous results when $(script)B$ is replaced by an infinite, conjugate symmetric subset B of $(script)B$, i.e., given such a set B is it true that for every s with $0 \leq s \leq \dim(B)$ there exists a conjugate symmetric set F contained in B such that $\dim(F) = s$?

The results in this lecture are joint work with Professor Richard S. Falk.

To determine a von Neumann algebra through its positive unit sphere

Prof. Ngai-Ching Wong
National Sun Yat-sen University, Taiwan

Abstract. We answer, in the affirmative, Tingley's problem for positive unit spheres of von Neumann algebras. More precisely, let $T : PS(A) \to PS(B)$ be a bijection between the sets of positive norm one elements of von Neumann algebras A and B. We show that if T preserves metric, then T extends to a Jordan *-isomorphism from A onto B. In fact, one can still conclude that A is Jordan *-isomorphic to B, when T satisfies a weaker assumption that it preserves the diametral relations, that is, for any positive norm one elements $a, b \in A$, one has $\|Ta - Tb\| = 1$ if and only if $\|a - b\| = 1$. We also show that A and B are Jordan *-isomorphic if there is an order isomorphism $S : P(A) \to P(B)$ between the projection lattices of A and B, that preserves also the diametral relation, that is, for any projections $p, q \in A$, $\|Sp - Sq\| = 1$ if and only if $\|p - q\| = 1$. If S is metric preserving, then S extends to a Jordan *-isomorphism from A onto B. Actually, the above results are proved in the slightly more general situation when A and B are AW*-algebras.

Prime counting formulas for Graph-Directed Self-Conformal Iterated Function Systems

Prof. Lars Olsen
University of St. Andrews, UK

Abstract. Prime counting formulas count the number of appropriate defined "prime elements" less than a given number, e.g. the number of prime numbers less than x or the number of prime geodesics whose length is less than x. We will develop a general theory of fractal and multifractal prime counting formulas for Graph-Directed Self-Conformal Iterated Function Systems. Several applications are presented, e.g. we will find explicit formulas for the q-th moment of self-conformal measures satisfying the Open Set Condition, and we will obtain a multifractal prime geodesic counting formula for classical Fuchsion Schottky groups.

On the mensuration of conformal fractals

Prof. Tushar Das
University of Wisconsin-La Crosse, USA

Abstract. Continued fractions have provided a natural playground for several advances in number theory, geometry, topology, dynamics, analysis, and probability theory. I will report on some dimension-theoretic research – https://arxiv.org/a/das_t_4.html –regarding the fascinating fractals that arise

from studying continued fractions and point to vistas among their conformal cousins about which much less is known. The talk will be accessible to anyone whose interests intersect a convex combination of functional analysis, dynamical systems, number theory, and fractal geometry.

Kleinian group actions and parabolic rational maps

Prof. Jonathan Fraser
University of St. Andrews, UK

Abstract. The Sullivan dictionary provides a conceptual framework to compare the actions of Kleinian groups and the dynamics of rational maps. Both settings generate interesting fractal sets (limit sets of Kleinian groups and Julia sets of rational maps). There is a particularly strong correspondence in the context of dimension theory. Restricting to the geometrically finite setting, in both cases there is a 'critical exponent' which returns the Hausdorff, box, and packing dimensions of the associated fractal, as well as the Hausdorff, packing, and entropy dimensions of the associated conformal measure. We show that, by slightly expanding the family of dimensions considered, a much richer theory emerges. This allows us to draw more nuanced comparisons, and provide novel discrepancies, between the Kleinian and rational map settings. This is joint work with Liam Stuart.

A note on the distance sets for graphs of functions

Prof. Amit Priyadarshi
IIT Delhi

Abstract. In this talk, we discuss about the famous "distance set conjecture" by Falconer and progress made in this direction so far. We discuss the Hausdorff dimensions of the distance sets and the difference sets for the graphs of continuous functions on the unit interval. We prove that the distance set conjecture is true for the class of graphs of continuous functions. We also prove that the distance set conjecture is valid for a dense subset of the class of graphs of bounded functions. We also determine a non-trivial lower bound for the upper box dimension of the difference set of a set in the plane and discuss the dimension of the distance set of the product of sets. This is a joint work with Manuj Verma.

Introduction of Fractional Calculus with application

Prof. Syed Abbas
IIT Mandi

Abstract. In this talk, we discuss the basic theory of fractional calculus. The application to integral equations will be discussed.

Surjective isometries of C*-algebras and their spectra

Prof. Antonio Jiménez Vargas
University of Almería, Spain

Abstract. The known duality of the Bloch space of complex-valued functions on the open complex unit disc $\mathbb{D}$ is addressed under a new approach with the introduction of the concepts of Bloch molecule and Bloch-free Banach space over $\mathbb{D}$. We introduce the notion of compact Bloch mapping from $\mathbb{D}$ into a complex Banach space X and study some properties: invariance by Möbius transformations, linearization from the Bloch-free Banach space over $\mathbb{D}$, factorization of their derivatives, Banach ideal property and transposition on the normalized Bloch space.
This is a joint work with David Ruiz–Casternado (University of Almería, Spain). This research was partially supported by Junta de Andalucía grant FQM194 and by Ministerio de Ciencia e Innovación grant PID2021-122126NB-C31 funded by MCIN/AEI/10.13039/501100011033 and by "ERDF A way of making Europe".

Surjective isometries of C*-algebras and their spectra

Prof. Dijana Ilišević
University of Zagreb, Croatia

Abstract: In this talk we describe the structure of surjective isometries of C*-algebras, both complex linear and real linear, following a long line of work starting with the celebrated Banach-Stone theorem. We also investigate the spectrum of periodic surjective isometry (classical in the complex case and complex spectrum in the real case) and give several illustrative examples. Recent results to be presented in this talk stem from joint work with Fernanda Botelho, joint work with Chih-Neng Liu, Bui Ngoc Muoi and Ngai-Ching Wong, and joint work with Catherine Bénéteau, Fernanda Botelho, María Cueto Avellaneda, Jill E Guerra, Sana Kazemi and Shiho Oi.

On unification of three extragradient methods via the property (A) for variational inequality problems with applications

Prof. D. R. Sahu
Banaras Hindu University

Abstract. The main strategy of this talk is intended to speed up the convergence of the inertial Mann iterative method and further, speedup it through the normal S-iterative method for a certain class of nonexpansive type operators that are linked with variational inequality problems. Our new convergence theory permits us to settle down the difficulty of unification of Korpelevich's extragradient method, Tseng's extragardient method, and subgardient extragardient method for solving variational inequality problems through an auxiliary algorithmic operator, which enjoys the following:

- the seed operator has the property (A) with respect to the algorithmic operator.

An interesting finding is that the relaxed inertial normal S-iterative extragradient methods do influence much more on convergence behaviour.

Some Extensions of the Banach Contraction Principle and Few Applications

Prof. Tanmoy Som
IIT(BHU)

Abstract. The Banach Contraction Principal (BCP) is one of the most important result of analysis, considered as the source of metric fixed point theory and perhaps it is the most widely applied fixed point result in many branches of mathematics. The BCP has been ex tended in many different directions. In fact, there is vast amount of literature dealing with extensions/generalizations of Banach Contraction Principle (BCP). It is nearly impossible to cover all the known extensions of the BCP. In this talk, we shall discuss some basic extensions of BCP for single and multi-valued maps in the setting of metric spaces. Precisely, we shall consider some selected extensions of the BCP for single-valued maps including the Caristi fixed point result in the setting of metric spaces. We also study multivalued versions of the BCP including Nadler's result and Reich problem. Some others extensions of BCP with respect to generalized distances will also be presented.

ON A SUBCLASS OF NORM ATTAINING OPERATORS

Prof. G. Ramesh
IIT Hyderabad

Abstract. Let H be a complex Hilbert space and B(H) be the space of all bounded linear operators on H. We say $T \in B(H)$ norm attaining if there exists $x \in H$ with $\|x\| = 1$ such that $\|Tx\| = \|T\|$. We define a new class $\beta(H) := \{T \in B(H) : T$ attains norm on every reducing subspace of $T\}$. In this talk we discuss properties and structure of elements in $\beta(H)$.

Research on fractal dimensions of the graph of continuous functions under operations

Prof. Yongshun Liang
Nanjing University of Science & Technology, China

Abstract. In this talk, we mainly discuss the changes in fractal dimensions of linear combinations of fractal functions. Firstly, when the Box dimension exists, we roughly divide it into three situations. Secondly, when the Box dimension does not exist, we have carefully divided it into more than ten different situations. These works have important supporting significance for our subsequent research on the changes in fractal dimensions of fractional calculus functions and fractal function approximation theory.

Integral Read-Bajraktarević Operators

Prof. Peter Massopust
Technical University of Munich, Germany

Abstract. We introduce the novel concept of integral Read-Bajraktarević (iRB) operator and discuss some of its properties. We show that this iRB operator generalizes the known Read-Bajraktarević (RB) operator and we derive conditions for the fixed point of the iRB operator to belong to certain function spaces.

TBA

Prof. Jugal K. Prajapat
Central University of Rajasthan

Abstract. TBA

TBA

Prof. Tapobrata Lahiri
IIIT Allahabad

Abstract. TBA

Symmetric fractional order reduction method with L1 scheme on graded mesh for time fractional nonlocal diffusion-wave equation of Kirchhoff type

Dr. Sudhakar Choudhary
IITRAM

Abstract. In this talk, we present a linearized fully-discrete scheme for solving a time fractional nonlocal diffusion-wave equation of Kirchhoff type. The scheme is established by using the finite element method in space and the L1 scheme in time.

Automorphisms and generalized projections on spaces of analytic functions

Dr. Rahul Maurya
IIT Kanpur

Abstract. In this talk, we present complete classifications of automorphisms of two closed subalgebras of the space of bounded analytic functions on the open unit disc $\mathbb{D}$, namely, the subalgebra of functions vanishing at the origin, and the subalgebra of functions whose first derivative vanishes at the origin. The later subalgebra is known as the Neil algebra. We also characterize generalized tri-circular projections on $H^p(\mathbb{D})$ and $H^p(\mathbb{D}^2), 1 \leq p \leq \infty, p \neq 2$. This is joint work with Prof. Jaydeb Sarkar and Dr. Aryaman Sensarma.

Approximation of Fixed Points, stability and data dependence results

Dr. Javid Ali
Aligarh Muslim University

Abstract. In this talk, we discuss a newly introduced two step fixed point iterative algorithm. We prove a strong convergence result for weak contractions. We also prove stability and data dependency of a proposed iterative algorithm. Furthermore, we utilize our main result to approximate the solution of a nonlinear functional Volterra integral equation. If time permits, then we will discuss Image recovery problem as well.

Theorem 1. Let $T : C \to C$ be a weak contraction satisfying (*), where C is a nonempty, closed and convex subset of a Banach space X. Then proposed iterative algorithm is almost T-stable.

An interplay between integrability and convergence in Banach spaces

Dr. Nisar Ahmad Lone
JK Institute of Mathematical Sciences

Abstract. We derive a relationship between Riemann integrability and different forms of convergence in a Banach spaces, and will try to establish a connection between integrability of a vector-valued function (a function taking values in an infinite dimensional Banach space) and the topological structure of a Banach space.

Linear maps preserving parallel matrices and triangle equality attaining pairs

Dr. Ya-Shu Wang
National Chung Hsing University, Taiwan

Abstract. Two (real or complex) $m \times n$ matrices A and B are parallel with respect to the operator norm $\|\cdot\|$ if $\max_{|\mu|=1} \|A + \mu B\| = \|A\| + \|B\|$. When $\|A+B\| = \|A\|+\|B\|$, we refer to A, B as a pair attaining the triangle equality.

In this talk, I will describe linear maps T on $m \times n$ matrices that preserve parallel pairs, meaning $T(A)$ and $T(B)$ are parallel whenever A and B are parallel. Additionally, I will provide a characterization of linear maps T on $m \times n$ matrices that preserve triangle equality attaining pairs, i.e., $T(A)$ and $T(B)$ form a pair attaining the triangle equality whenever A and B do.

On Hahn-Banach theorem in Banach spaces

Dr. Tanmoy Paul
IIT Hyderabad

Abstract. A bounded linear functional on a subspace has an extension to the space that preserves the norm, is a well-known fact. In the literature on Banach spaces, several investigations concerning this extension property have been explored in recent years. A few milestones in this direction are; Bilinear form of Hahn-Banach extension, Invariant Hahn-Banach extension, Connections with Banach-Tarski paradox, Uniqueness of Hahn-Banach extension, Operator version of Hahn-Banach extension and many more. Robert Phelps was the first who initiated the investigations on the Uniqueness of Hahn-Banach extension in [1], which is referred to as property (U). We encounter property (U) in various Banach spaces and in their subspaces. Study on the operator version of Hahn-Banach theorem is another aspect of this talk. It is well-known that a bounded extension may not exist for a bounded linear operator between normed spaces. We introduce spaces with this property which are referred to as Injective spaces. We will encounter some spaces where bounded extension exists but $\|\tilde{T}\| \leq \lambda \|T\|$ for some $\lambda > 1$, known as P_λ spaces.
References: [1] R.R.Phelps, Uniqueness of Hahn-Banach extensions and unique best approximation. Transactions of the American Mathematical Society, 95(2), 238-255 (1960).

Basic Information on Mathematical and Epidemiological Modelling

Dr. Sunil Kumar
NIT Jamshedpur

Abstract. The tumor is very serious and dangerous disease in medical science and it is defined as a mass or lump of tissue formalized by the aggregation of abnormal cells. In just a few days, it has become important to try to understand diseases with high mortality rates around the world, such as infectious diseases and cancer. A well-known non integer order tumor-immune model is being used to study the dynamical behaviour. We present in this research paper a comparative and chaotic investigation of tumor and effector cells using a non-integer order tumor-immune dynamic model. We look at the interactions between different tumor cell populations and immunological composition using a model of a real world medical research problem. Further, we conclude the stability of the non integer order cancer model. Our findings will be valuable to biologists in the treatment of cancer.

Distinguished varieties in the polydisc and dilation of commuting contractions

Dr. Sourav Pal
IIT Bombay

Abstract. A distinguished variety in the polydisc $\mathbb{D}^n$ is an affine complex algebraic variety that intersects $\mathbb{D}^n$ and exits the domain through the n-torus $\mathbb{T}^n$ without intersecting any other part of the topological boundary of $\mathbb{D}^n$. We show two different characterizations for a distinguished variety in the polydisc in terms of the Taylor joint spectrum of certain linear matrix-pencils and thus generalize the seminal work due to Agler and McCarthy on distinguished varieties in $\mathbb{D}^2$. We show that a distinguished variety in $\mathbb{D}^n$ is a part of an affine algebraic curve which is a set-theoretic complete intersection. We also show that if $(T_1, \ldots, T_n)$ is commuting tuple of Hilbert space contractions such that the defect space of $T = \prod_{i=1}^n T_i$ is finite dimensional, then $(T_1, \ldots, T_n)$ admits a commuting unitary dilation $(U_1, \ldots, U_n)$ with $U = \prod_{i=1}^n U_i$ being the minimal unitary dilation of T if and only if some certain matrices associated with $(T_1, \ldots, T_n)$ define a distinguished variety in $\mathbb{D}^n$

A norming property of subspaces of a Banach space

Dr. Debmalya Sain
IIIT Raichur

Abstract. The purpose of this talk is to discuss the newly introduced notion of m-heavy subspaces of a Banach space, from the perspective of norm attainment. The importance of m-heavy subspaces in understanding the analytic and geometric structures of the concerned space and its dual will be illustrated through some interesting properties possessed by these subspaces, along with concrete examples. Several applications of m-heavy subspaces, including the well-studied notion of Hahn-Banach extension operators will be discussed in order to establish the specialty of these subspaces.

Higher-order spectral shift measures in several variables

Dr. Arup Chattopadhyay
IIT Guwahati

Abstract. In recent years, higher-order trace formulas of operator functions have attracted considerable attention to a large part of the perturbation theory community. In this talk, I will discuss estimates for traces of higher-order derivatives of multivariable operator functions with associated scalar functions arising from multivariable analytic function space and, as a consequence, derive

higher-order spectral shift measures for pairs of tuples of commuting contractions under Hilbert-Schmidt perturbations. These results sub stantially extend the main results of [Trace Formulas for Multivariate Operator Functions, Integr. Equ. Oper. Theory. 81 (2015), no. 4, 559–580], where the estimates were proved for traces of first and second order derivatives of multivariable operator functions. This is a joint work with Chandan Pradhan and Saikat Giri.

Ball proximity of M-ideals of compact operators

Dr. C. R. Jayanarayanan
IIT Palakkad

Abstract. In this lecture, we will discuss the proximinality of closed unit ball of M-ideals of compact operators on Banach spaces. We will show that every positive (self-adjoint) operator on a Hilbert space has a positive (self-adjoint) compact approximant from the closed unit ball of space of compact operators. This is a joint work with Sreejith Siju.

Characterization of amenability in terms of properties of the Orlicz Figa-Talamanca Herz algebras

Dr. Rattan Lal
Punjab Engineering College

Abstract. We will talk about some results in the Orlicz Figa-Talamanca Herz algebras.